Engineering Notes

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Libration Damping of a Tethered Satellite by Yo-Yo Control with Angle Measurement

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Introduction

LIBRATION damping during stationkeeping of a tethered satellite [1-4] has been proposed using yo-yo control with tether tension measurement.[5-7] However, tension for short tethers is very small (of the order of an ounce for a 10-km tether), and the variation of tension with libration, which must be measured, is much smaller and requires sophisticated and costly instrumentation. Accordingly, other feasible ideas such as control of deployment rate have recently been proposed.[8] An alternate idea of libration control that does not require the measurement of tether tension is due to Rheinfurth in Ref. 2, and involves reeling the tether in and out at appropriate times – damping the libration in a way that is the reverse of pumping a playground swing.

In this note we extend this idea by exploiting the special features of an orbiting pendulum during in-plane and out-of-plane motion. In particular, we show that a substantial improvement can be made in damping in-plane libration over that predicted in Ref. 2. We also have developed a concomitant length control law for stationkeeing. The basic assumption we use is that the libration angles and (derived) rates can be measured.

Previous studies[1,3] of the motion of a gravity gradient stabilized tether have shown that at intermediate altitudes (above about 500 km) where the effect of air drag is negligible, the gravity gradient forces are adequate to keep the tether quite straight (even considering other forces such as solar pressure, etc.). Hence, the angle of the tether as it leaves the parent satellite is a good indication of the angle of the line of sight to the end mass of the tether. The angle of the tether relative to the primary satellite is measured using a speaker-coil-type sensor around the tether at the point of attachment. And, with knowledge of the orientation of the primary satellite to local vertical, the orientation of the initial length of the tether is known. The angular rate of the libration is derived from continuous knowledge of the angle.

Dynamical Model

The dynamics of a gravity gradient stabilized tethered satellite system have been studied extensively.[2,4] Here we consider an idealized model that represents, in bare essence, such a system. Figure 1 illustrates the configuration being addressed. The attachment point for the tether is assumed to move in a circular orbit with an angular velocity ω. The mass of the tether and its payload is lumped as a particle at the end of a massless, rigid rod. Rotations through an orbit in-plane angle θ, followed by an orbit out-of-plane angle ϕ, describe the orientation of the tether with respect to an orbiting triad with $\boldsymbol{r}$ along the vertical, $\boldsymbol{v}$ along the velocity vector, and $\boldsymbol{n}$ along orbit normal; the instantaneous length of the tether is L. For prescribed tether deployment rate $\dot{L}$, the equations of motion for this system are

$$\ddot{\theta} = -2\dot{L}(\omega + \dot{\theta})/L + 2(\omega + \dot{\theta})\dot{\phi}\tan\phi - 3\omega^2 \sin\theta\cos\theta \quad (1)$$

$$\ddot{\phi} = -2\dot{L}\dot{\phi}/L - (\omega + \dot{\theta})^2 \sin\phi\cos\phi - 3\omega^2\cos^2\theta\sin\phi\cos\phi \quad (2)$$

These equations are used to simulate the action of the deployment rate control law to be described in the following sections. For the tether to always remain taut, it is necessary that tether tension T be always positive, where T is given by

$$T = L[\dot{\phi}^2 + (\omega + \dot{\theta})^2\cos^2\phi + \omega^2(3\cos^2\theta\cos^2\phi - 1)] - \ddot{L} \quad (3)$$

This places a constraint on the maximum value of the rate of change of the deployment rate $\ddot{L}$. Specifically, we see from Eq. (3) the necessary condition that

$$\ddot{L} < 3\omega^2 L$$

Although the control law developed in this paper is evaluated in the simulation using the nonlinear model based on Eqs. (1-3), design of the actual control comes from an understanding of the libration behavior of the tethered satellite about the

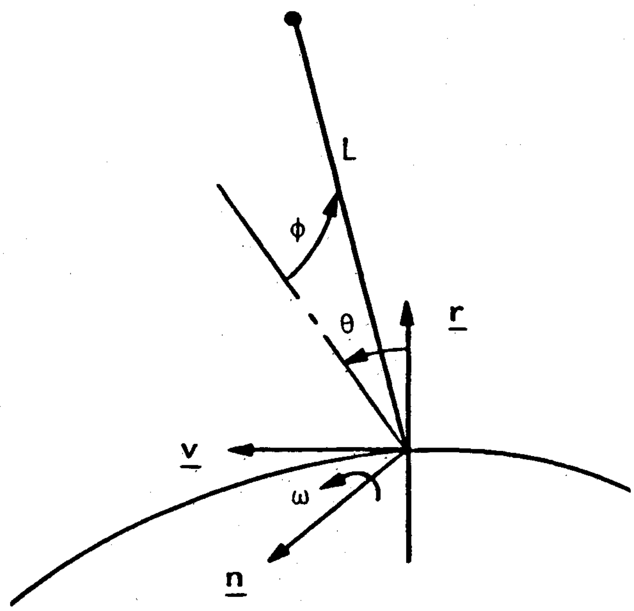

Fig. 1 System configuration.

Received May 19, 1988; revision received July 25, 1988.

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equilibrium condition, i.e., for small-angle excursions about the local vertical.

For prescribed deployment and retraction rates, the linearized equations of motion for small angles are

$$\ddot{\theta} = -2\dot{L}(\omega + \dot{\theta})/L - 3\omega^2\theta \tag{4}$$

$$\ddot{\phi} = -2\dot{L}\dot{\phi}/L - 4\omega^2\phi \tag{5}$$

For damping librations about the local vertical by varying the tether length, the linearized equations of motion, Eqs. (4) and (5), show that the in-plane motions are more sensitive to length variations than are out-of-plane motions, since the multiplier of $\dot{L}$ in Eq. (4) is typically 10 times greater than the multiplier of $\dot{L}$ in Eq. (5).

Consider first the roll libration motion. At the point of maximum roll angle, the roll rate $\dot{\phi}$ is zero. This suggests that the tether could be either deployed or retracted without affecting the roll angular motion (since the multiplier of $\dot{L}$ is zero). On the other hand, when the roll angle is zero (at the local vertical), the roll angular rate is maximum, and a change in tether length at this time has maximum effect on the roll momentum and energy. Since lengthening the tether removes energy by decreasing the roll rate, a roll damping effect could be accomplished by the strategy of lengthening the tether as it passes through local vertical and then shortening the tether the same amount at the maximum angle (zero roll rate) position. This scheme decreases, or damps, the roll libration at each zero, passing by an amount proportional to the roll rate and to the change in the tether length. Inspection of Eq. (5) shows that a 10% increase in the tether length will decrease the roll rate by 20%. The action described for roll damping is exactly analogous to the pumping of a swing. In this case we are pumping it down rather than pumping up.[2] The concept is illustrated in Fig. 2.

The situation for pitch is quite different. Here we have dynamics that are similar to the spinning ice skater. This is due to the effect of the (orbital) angular motion in the pitch direction necessary to keep the tether aligned along the local vertical. This in effect means that for pitch rates that are small compared to orbital rate (i.e., of the order of 10% or so), change in tether length will affect pitch rate about the same regardless of the pitch angle because of the effect of the high orbital rate. As it turns out, great advantage can be taken of this fact.

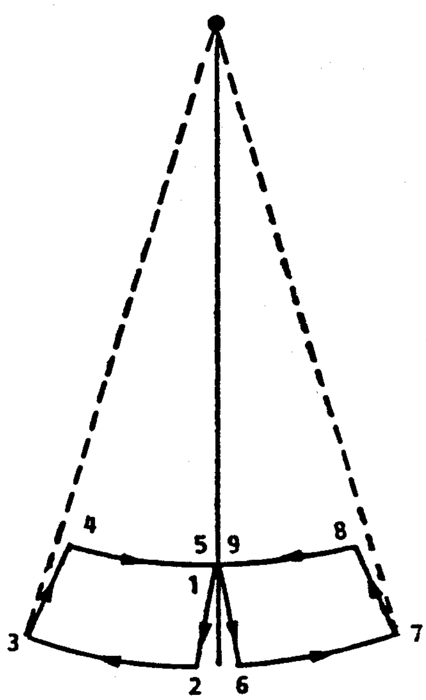

Fig. 2 Control of out-of-plane angle.

In particular, if the tether length and, hence, pitch rate is changed by an appropriate amount at the time the pitch angle is zero (along the local vertical in-plane), the pitch libration can be completely eliminated. Specifically, if the maximum pitch rate is 10% of the orbital rate, then at the time the tether is going through zero pitch angle in the same direction as the orbital rate, the tether can be increased in length by 5% and the total pitch rate will be reduced to the orbital rate, i.e., the relative pitch rate will be reduced to zero. Since this action is taken at the time of pitch angle zero, the pitch libration is zeroed in one stroke. (If the pitch rate is going in the opposite direction from the orbital rate at the time of zero angle, then the tether length should be decreased to increase the pitch rate and eliminate the libration.) The magnitude and direction of the deployment is determined by equating the pitch angular momentum before and after deployment (since there are no external torques). Thus

$$mL^2(\omega + \dot{\theta}) = (mL^2 + 2mL\ dL)\omega \tag{6}$$

where dL is the small finite change in length. This leads to the result:

$$2\omega\ dL = L\dot{\theta} \tag{7}$$

The sign and magnitude of $\dot{\theta}$ determines the sign and magnitude of dL. The pitch control concept is illustrated in Fig. 3a for the case in which the pitch rate $\dot{\theta}$ is in the same direction as orbital angular velocity ω, thus requiring a deployment.

These facts from the dynamics suggest a control scheme that damps the roll libration to an acceptable amplitude first and then removes the pitch libration in one shot. This sequence is used because the process of damping roll continually perturbs (pumps) pitch even if it started with no pitch libration. (This cannot be avoided by synchronizing pitch and roll as their libration periods are not commensurate.) The pumping effect is illustrated by the simulation run of Fig. 4 in which the control scheme is invoked with initial pitch angle and rate set equal to zero. On the other hand, once roll is sufficiently damped, lengthening or shortening the tether for pitch control has little effect on roll since the effect is proportional to the roll rate (which is now small).

One other point requires attention. After angular librations have been damped, the tether may be left at a length that is not optimum. This suggests a final length control to a desired length. This is implemented by taking advantage of the pitch dynamics. Once the roll and pitch are at acceptably small angles, the action taken is to simply reduce the error in length by half. This will induce a pitch motion by immediately creating a pitch rate. If the tether length is too long by, say, 100 m, shortening it by 50 m would cause the pitch rate to increase in the direction of the orbital rate. Exactly one-half of a pitch libration period later the pitch rate would be the same, but in the opposite direction, and the pitch angle would again be at, or near, zero. At this point, engaging the pitch control law would cause the length to shorten again by exactly the same 50 m in order to remove the residual (unwanted) pitch rate and also leave the tether at the desired length. Figure 3b shows the process for the case where the initial length is longer than the desired length.

Implicit in the preceding scheme is the assumption that the tether be deployed or retracted in a time that is short in comparison to the libration period. It is also necessary that the tension never become negative. To accomplish the latter we set or command the deployment acceleration for all rate changes to

$$\ddot{L} = 2.25\omega^2 L \tag{8}$$

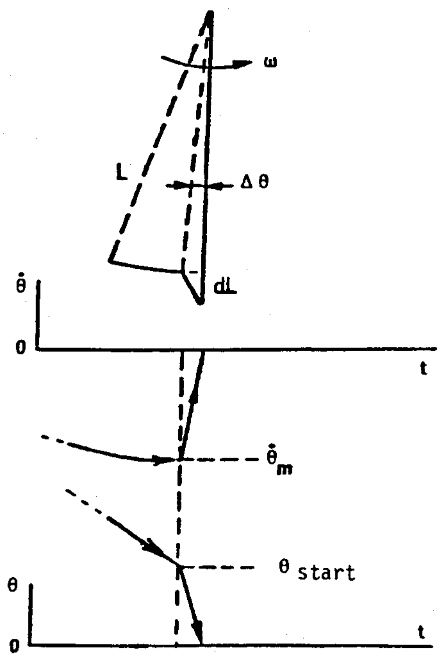

Fig. 3a Control of in-plane angle.

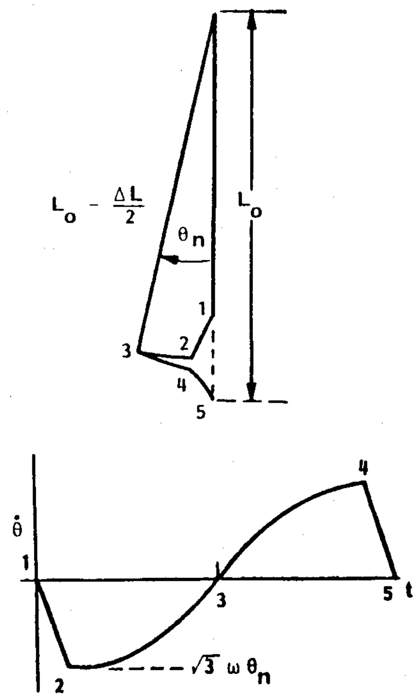

Fig. 3b Length control.

The coefficient 2.25 guarantees that tension in Eq. (3) never gets negative for libration angles as great as 20 deg in both pitch and roll simultaneously. This constraint is implemented by integrating $\ddot{L}$ to obtain $\dot{L}$, and by using appropriate initial conditions. Two different cases arise. For deployment, $\dot{L}$ must start out at zero and may increase as high as needed following the prescribed acceleration. Rapid nulling of a high positive $\dot{L}$ (of the order of a couple of meters per second) in a second or so presents no problem, as it causes a negative $\ddot{L}$ that does not violate the constraint that $\ddot{L} < (+\,3L\omega^2)$. Specifically, we have for $\dot{L}$ during deployment the following:

$$\dot{L} = 1.5\omega[dL + (2L\,dL)^{1/2}] \tag{9}$$

where $dL = L - L_{\text{Initial}}$, and we have used the first three terms in the series for $e^{1.5\omega t}$.

For retraction, the opposite situation prevails. That is, $\dot{L}$ is started with a high retraction rate and then is caused to slow down gradually by the prescribed $\ddot{L}$ amount. In this case we have for $\dot{L}$

$$\dot{L} = -1.5\,\omega[dL + (2L\,dL)^{1/2}] \qquad \text{for length changes} \tag{10}$$

where $dL = L_{\text{Final}} - L$, or, using Eq. (7)

$$\dot{L} = -1.5\,L[\dot{\theta}/2 + (\omega\dot{\theta})^{1/2}] \qquad \text{for } \dot{\theta} \text{ changes} \tag{11}$$

We guarantee adherence to the constraint of no negative tension by calling for a stop of the simulation if tension becomes negative. In all simulations we have run, tension has never become negative.

Deployment (retraction) at the aforementioned rates allows deployment of 100 m of tether in about 250 s, which is small compared to orbit period (e.g., about 6400 s for a 600 n.mi. altitude). This is important in that the basic assumption in the control scheme is that the length changes occur while the libration angles are at (or near) their zero or peak values. The only consequence of the finite time for length change is that pitch librations are not reduced to zero. They are, however, reduced to well within the desired minimum.

Control Logic

The control laws are constructed on the premise that roll, pitch, and length would be controlled independently (with final length control making use of the pitch control already in place). Roll control is effected by the simple expedient of deploying and retracting the tether between fixed limits at the appropriate time. Retraction is commanded when the roll rate drops below a small threshold of the order of 0.001 times orbital rate, indicating the occurence of maximum roll angle and zero roll rate. Deployment is invoked when the roll angle is less than 0.001 rad, which is small in comparison to a desired maximum steady-state roll libration angle of 0.01 rad. For a nominal tether length of 1000 m as used in the case simulated, the length limits are set at 950 and 1050 m, causing the roll damping to displace the tether length no more than 50 m from the nominal length. As long as the roll libration exceeds a barrier based on a metric determined from the energy in the roll motion, pitch and length control are bypassed. This metric is used since a roll rate barrier fails at peak angles, and an angle barrier fails at peak rate. Respectively, the metric and barrier are the energy-like measures:

$$E_\phi = \dot{\phi}^2 + 4\omega^2\phi^2 \tag{12a}$$

$$E_b = 4\omega^2\phi_m^2 \tag{12b}$$

where $\phi_m = 0.01$ rad, the maximum desired residual angle, ω is the orbital angular rate, ϕ the instantaneous roll angle, and $\dot{\phi}$ the roll rate. Selection of this quantity is based on the premise that the roll motion is a gravity gradient libration whenever the tether is not deploying or retracting. The roll bound E_b is obtained by setting $\dot{\phi}$ to zero in E_ϕ and using the maximum desired, or damped, roll libration angle ϕ_m for ϕ.

Pitch control is allowed to function as soon as the quantity E_ϕ falls below the bound E_b. This occurs as pitch approaches its first zero crossing after the completion of roll damping. At that point the tether is either deployed or retracted depending on whether the pitch rate is in the direction of the orbital rate or counter to it. What is desired is that the pitch angle be zero at the same time that pitch rate is brought to zero. This is accomplished by starting the pitch deployment just prior to zero at an angle that varies, as follows, with the ratio of pitch rate to orbital rate, i.e.,

$$\theta_{\text{start}} = 0.222(\dot{\theta}/\omega)^{3/2} \tag{13}$$

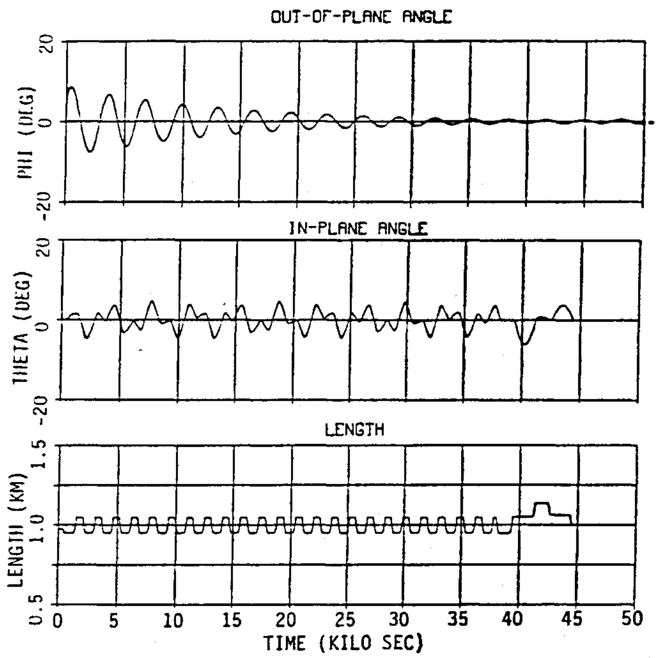

Fig. 4 Capture for initial pitch = 0.

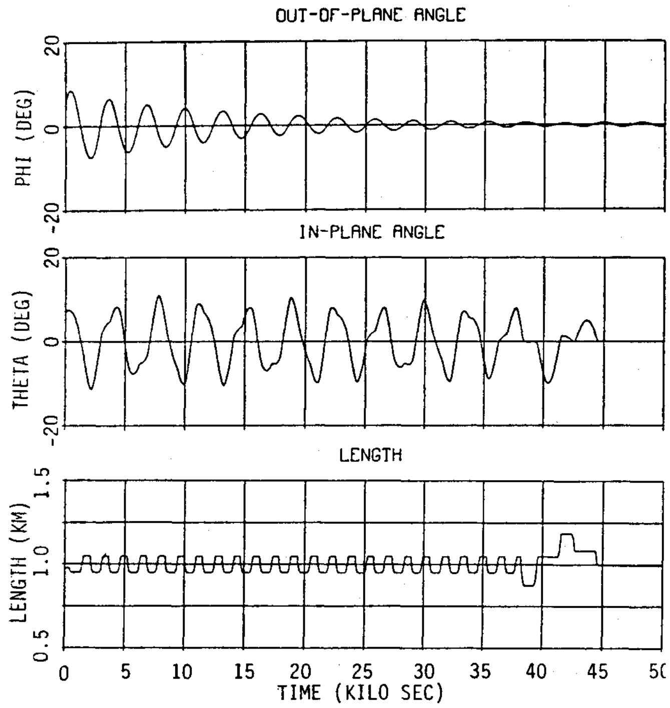

Fig. 5 Typical system capture example.

Because retraction of the tether can potentially pump the roll motion, it is necessary to take special care with the pitch control whenever retraction is called for. We do this by simply not starting a pitch retraction mode if the roll rate is so large that the pitch correction would increase the roll rate to the maximum desired roll rate. For those situations where the roll limit would be exceeded, pitch control is delayed one-half of a pitch libration cycle. At this point the control calls for lengthening instead of shortening the tether to eliminate pitch (and damps roll further in the process). At the conclusion of pitch damping, both roll and pitch librations are at an acceptably low level, and the tether is at some arbitrary length, the exact value of which is a function of the specific initial conditions.

As previously outlined, changing the tether length in two equal steps separated by one-half of a pitch libration period puts the system in the final desired state. However, a change in tether length immediately affects pitch rate. Therefore, because of the finite time required to accomplish length control, the pitch control must be locked out while the length change is being effected, i.e., until the length has been brought the appropriate half-distance toward the final desired state.

It is observed that when pitch is in a gravity gradient libration state the pitch rate and the pitch angle are of the opposite sign at deployment initition. On the other hand, when tether length control is in effect, deployment causes both pitch rate and pitch angle to change by the same sign! Thus, the strategy used is to bypass pitch control whenever the product of pitch rate and pitch angle is greater than zero. This allows the length control to complete its job independently of the pitch control and forces the next implementation of the pitch control to delay (typically for one-half of a pitch libration period) until the sense of the angle and the rate are opposite.

The length control must be locked out whenever pitch damping is being effected. This is accomplished simply by setting a flag so that the length control is bypassed while the pitch control action is on.

As in the case of damping pitch, retraction for length control also can pump roll. Since length retraction could be called for when the roll rate is high, a constraint on initiation of a length retraction is invoked. Specifically, if the roll rate that would result after retraction is greater than the maximum desired roll rate, the length retraction is inhibited until roll rate is sufficiently small. This delays the final length adjustment, but guarantees that the final roll angle is sufficiently small.

Simulation Results

The preceding control logic is implemented in Fortran and used in a simulation of the tether dynamics. To test the effectiveness of this nonlinear control system, it is appropriate to run cases with several different initial conditions. Simulations are made for nonzero roll, roll rate, pitch, pitch rate, and nonnominal initial tether length. For initial roll angles of the order of 0.2 rad, times of the order of 7 orbit periods are necessary to reduce roll by a factor of 20 (to 0.01 rad). For a dL/L ratio of 0.1, this gives a roll damping factor of $\zeta = 0.0495$, less by 22% than the number predicted by Rheinfurth in Ref. 9. Figure 5 shows the results of a typical system capture sequence for stationkeeping of a 1-km-long tether with a payload deployed out of a primary satellite orbiting at a 600 n.mi. altitude. These results are representative of several runs made with different initial conditions established by advancing the phase of the initial roll angle by 1/16 of an orbit period. This technique is used because it is found that the final (endgame) pitch and length control details are strongly correlated to the relative phase between pitch and roll at the initiation of the pitch control. Results of initial roll conditions separated by 180 deg are identical to one another.

Conclusions

Libration damping of a tether system operating in a gravity gradient mode has been simulated and shown to be effective. Simultaneous roll, pitch, and length perturbations of the order of 10% are damped, or eliminated, within six to seven orbit periods. The actual length of time is largely dependent on the initial roll perturbation and requires about one orbit period for a 20% reduction using nominal (less than 10%) length variations. Pitch and length control are coupled and can be effected in less than one orbit period. The control law set forth in this paper is designed on the basis of a simple model, and needs to be verified with a model that accounts for tether mass and flexibility.

Acknowledgment

The authors would like to thank a reviewer for pointing out the restriction on the tether deployment acceleration that led to a realistic implementation of deployment rates.

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Functional Analysis Methods in the Study of the Optimal Transfer

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Introduction

IN the present Note the motion along the Earth and moon orbit in a given time of a space vehicle (acted upon by a low thrust) starting at the collinear libration points is analyzed. Let the interval $[0, T]$ be the duration of the vehicle motion and consider a linear space of the functions continuous on $(0,T)$. This space E consisting of L_1 $(0, T)$ absolutely integrable functions of the matrix of the fundamental solution for the uncontrolled system in a Banach space. The dual space E^* is the space of functionals, linear and continuous on $(0, T)$, representing the control vector. In E^* a norm is defined. Since the operator defined for the optimal transfer is bounded, the norm of this operator is the lower bound of the numbers satisfying the inequality given by the boundedness condition. Use is made of final condition for the considered problem to determine the control vector and the commutation time of the control.

Transfer in Vacuum

For a space vehicle acted upon by a low-thrust propulsion force, the transfer from the liberation points to the Earth or moon orbits is equivalent to a problem of encounter in orbits.

Received May 11, 1988; revision received Nov. 7, 1988.

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Considering a rotating system having its origin at the collinear libration points for the Earth-moon system the linearized system of equations of motion of the space vehicle can be written in the form given in Eqs. (1).

$$\dot{x}_1 = x_2 \tag{1a}$$

$$\dot{x}_2 = K_1 x_1 + 2\omega x_4 + u_1 \tag{1b}$$

$$\dot{x}_3 = x_4 \tag{1c}$$

$$\dot{x}_4 = -2\omega x_2 + K_2 x_3 + u_2 \tag{1d}$$

where x_1, x_3 represent the coordinates of the space vehicle and x_2, x_4 are the velocity components; (u_1, u_2) being the thrust acceleration component in the rotating system.

The solution of the system represented by Eqs. (1) is written in the form

$$x_i(t) = \sum_{j=1}^{4} R_{ij}(t)x_j(0) + \sum_{j=1}^{4} \int_0^t K_{ij}(t,\tau)u_j(\tau)\,\mathrm{d}t \tag{2}$$

where $R_{ij}(t)$ is the matrix of the normal fundamental solutions at $t=0$ of the system [Eqs. (1)] at $u_j = 0$. It follows

$$R_{ij}(0) = \delta_{ij} \tag{3}$$

We also denoted

$$K_{ij}(t,\tau) = \sum_{m=1}^{4} R_{im}(t)R_{mj}^{-4}(\tau) \tag{4}$$

R_{mj}^{-4} being the elements of the inverse fundamental matrix.

Functional Analysis Methods in Optimal Control

Let $[0, T]$ be the time interval on which we want to analyze the optimal transfer from the manifold

$$S_I = \{x \mid x_i(0) = x_I^i = 0 \qquad i = 1, \ldots 4\} \tag{5}$$

to the manifold

$$S_F = \{x \mid x_2(T) = c_2;\ x_4(T) = c_4\} \tag{6}$$

Let us consider the set

$$E = \{g \mid g = \{g_i(t)\} = [K_{i1}(T,t),\ K_{i2}(T,t)], \qquad i = 1, \ldots 4;\quad g_i \epsilon L_1(0,T)\} \tag{7}$$

Since $g_i\, L_1(0,T)$ is a Banach space it follows that E is a Banach space. Let us define in E the norm

$$\left\| g \right\| = \sum_{j=1}^{2} \int_0^T \left| g_j(t) \right| \tag{8}$$

The dual space of the linear and continuous functionals given by

$$E^* = \{u \mid u = \{u_j(t)\}; \qquad j = 1,2; \qquad u_j \epsilon L_\infty(0,T)\} \tag{9}$$

is also a Banach space that can be normalized by introducing the norm

$$\| u \| \max_{1\le j\le 2}\ \max_{0\le t\le T} |u_j(t)| \tag{10}$$

If $g\epsilon E$ and $u\epsilon E^*$ we may write

$$(g,u) = \sum_{j=1}^{2} \int_0^T g_j(t)u_j(t)\,\mathrm{d}t \tag{11}$$